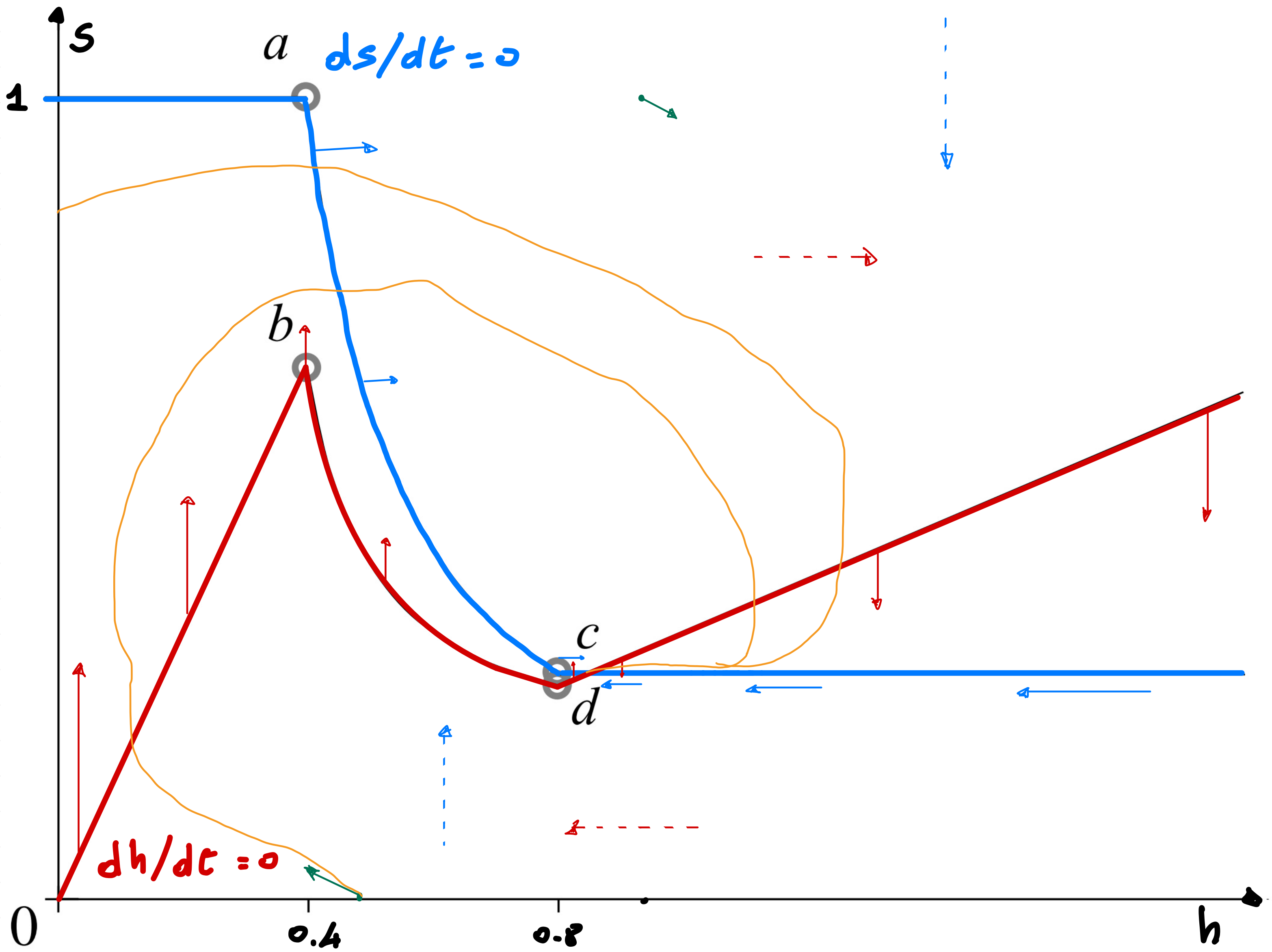
Exercise 1



d) Noting that

$$\frac{ds}{dt} = 0 \iff S = \frac{2.8}{1 + 9\, g(h)}$$

the top curve is the s-nullcline.

(see plot!)

b) On the s-nullcline:

$$a = (0.4, 1)$$

$$c = (0.8, 0.28)$$

On the h-nullcline:

$$\frac{dh}{dt} = 0 \iff -h + S_0 \, s \, g(h) = 0$$

For b we know $h = 0.4$:

$$-0.4 + 3 \, s \, 0.2 = 0$$

from which follows

$$b = (0.4, 2/3)$$

For d we know $h = 0.8$:

$$-0.8 + 3 s = 0$$

from which follows

$$b = (0.8, 0.27)$$

c) $(0.5, 0)$

$$\frac{dh}{dt} = -\frac{0.5}{5} = -0.1$$

$$\frac{ds}{dt} = \frac{2.8}{50} = 0.056$$

(see plot!)

d) (see plot!)

e) $(1, 1)$

$$\frac{dh}{dt} = \frac{-1+3}{5} = 0.4$$

$$\frac{ds}{dt} = -\frac{7.2}{50} = -\frac{14.4}{100} = -0.144$$

f) (Done after the g set, as suggested.
Fixed point is stable!)

(See plot!)

g1)

Nullclines are (assuming $g(h)=1$)

$$-h + J_0 s = 0$$

$$2.8 - 10 s = 0$$

Hence

$s = 0.28$ (Always, regardless of h as long as $h > 0.8$)

$h = 0.84$

Fixed point is at $(0.84, 0.28)$

g2)

$$J = \begin{pmatrix} -\frac{1}{\tau} & \frac{J_0}{\tau} \\ 0 & -\frac{10}{\tau_s} \end{pmatrix}$$

$$\det(J - \lambda I) = \left(-\frac{1}{\tau} - \lambda\right)\left(-\frac{10}{\tau_s} - \lambda\right) - 0 =$$

$$= \left(\lambda + t^{-1}\right)\left(\lambda + 10\, t_s^{-1}\right) =$$

$$= \lambda^2 + \left(10\, t_s^{-1} + t^{-1}\right)\lambda + 10\, t^{-1}\, t_s^{-1} = 0$$

$$\lambda = \frac{\left(-10 t_s^{-1} - t^{-1}\right) \pm \sqrt{\left(10 t_s^{-1} + t^{-1}\right)^2 - 40\, t^{-1} t_s^{-1}}}{2}$$

g3) $t^{-1} = 0.2 \quad t_s^{-1} = 0.02$

$$\lambda^{+} = \frac{(-0.2 - 0.2) + \sqrt{(0.2 + 0.2)^2 - 40 \; 0.2 \cdot 0.02}}{2}$$

$$= \frac{-0.4 + \sqrt{0.16 - 0.16}}{2} = -0.2$$

Fixed point is stable as both eigenvalues have negative real component.

94) To the limit $t_s \to 0$, $t_s^{-1} \to \infty$

$$\lambda^{+} \longrightarrow \frac{-10\, t_s^{-1} + \sqrt{100\, t_s^{-2} - 40\, t_s^{-1}}}{2} =$$

$$\frac{-10\, t_s^{-1} + 10^{-}\, t_s^{-1}}{2} = 0^{-}$$

To the limit $t_s \to \infty$, $t_s^{-1} \to 0$

$$\lambda^{\pm} \longrightarrow \frac{-t^{-1} \pm \sqrt{t^{-2} - 40\, t^{-1}}}{2} =$$

Fixed point stays stable!

$$\frac{-0.2 \pm \sqrt{0.04 - 40 \cdot 0.2}}{2} =$$

$$\frac{-0.2 \pm \sqrt{0.04 - 8}}{2} \cong$$

$$-0.1 \pm \sqrt{2}\, i$$

h) No. Increasing J_0 shifts the fixed point to the right, but $g(h)$ stays the same, eigenvalues stay the same, fixed point remains stable!

Because there is a zero in the off diagonal of J!

Exercise 2

a)

$$I(t_0) = \sum_n^N \sum_g \gamma(t_0 - t_g^n)$$

$$= \sum_n^N \sum_g w_0 \left[e^{-(t_0 - t_g^n)/t_2} - e^{-(t_0 - t_g^n)} \right]$$

$$I_0(t_0) = \int_{-\infty}^{t_0} J_0 \left[e^{-(t_0 - t)/t_1} - e^{-(t_0 - t)/t_1} \right] A(t)\, dt$$

b) A single spike produces

$$q = \int_0^\infty \gamma(t)\, dt =$$

$$= w_0 \int_0^\infty e^{-t/t_2} - e^{-t/t_1} = w_0 (t_2 - t_2)$$

Assuming A is a constant, I_{syn} is also constant:

$$I_{syn} = N A w_0 (\tau_2 - \tau_1)$$

$$= A J_0 (\tau_2 - \tau_1)$$

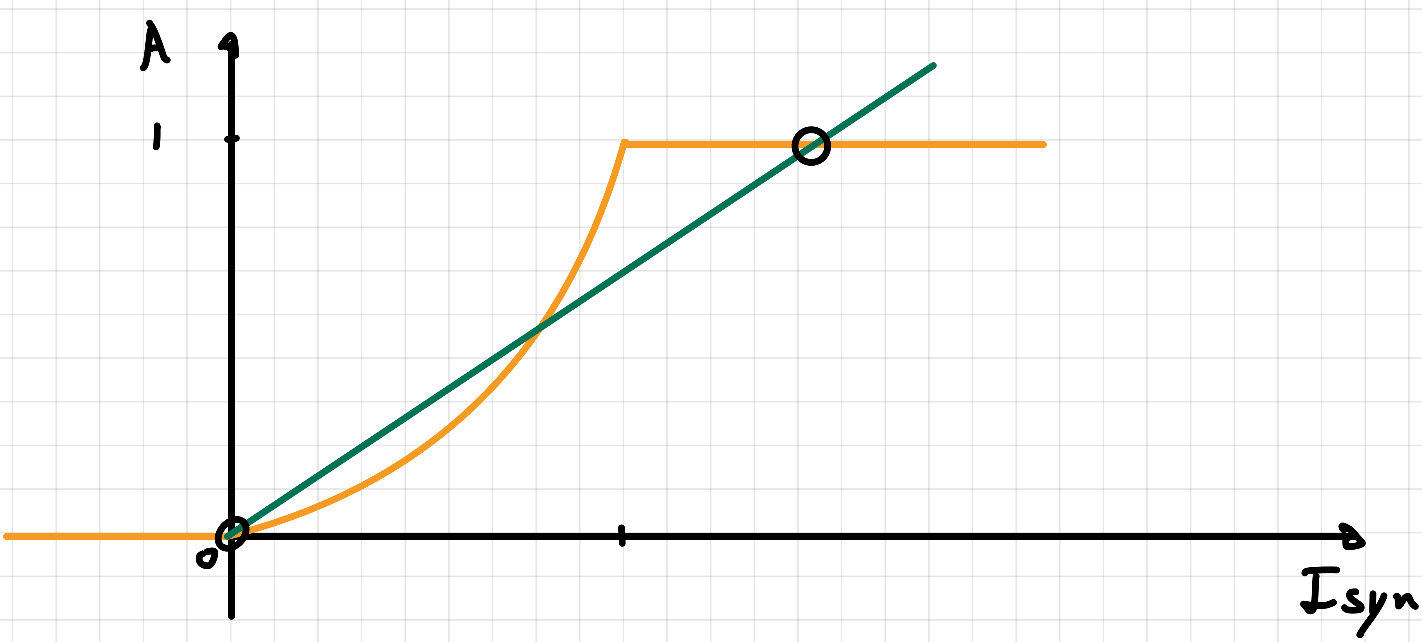
Plugging in the values:

$$I_{syn} = \frac{3}{2} A \quad \Rightarrow \quad A = \frac{2}{3} I_{syn}$$

At the limit $N \to \infty$, $A(t)$ converges to $\nu(t)$, from which we get (assuming constant I):

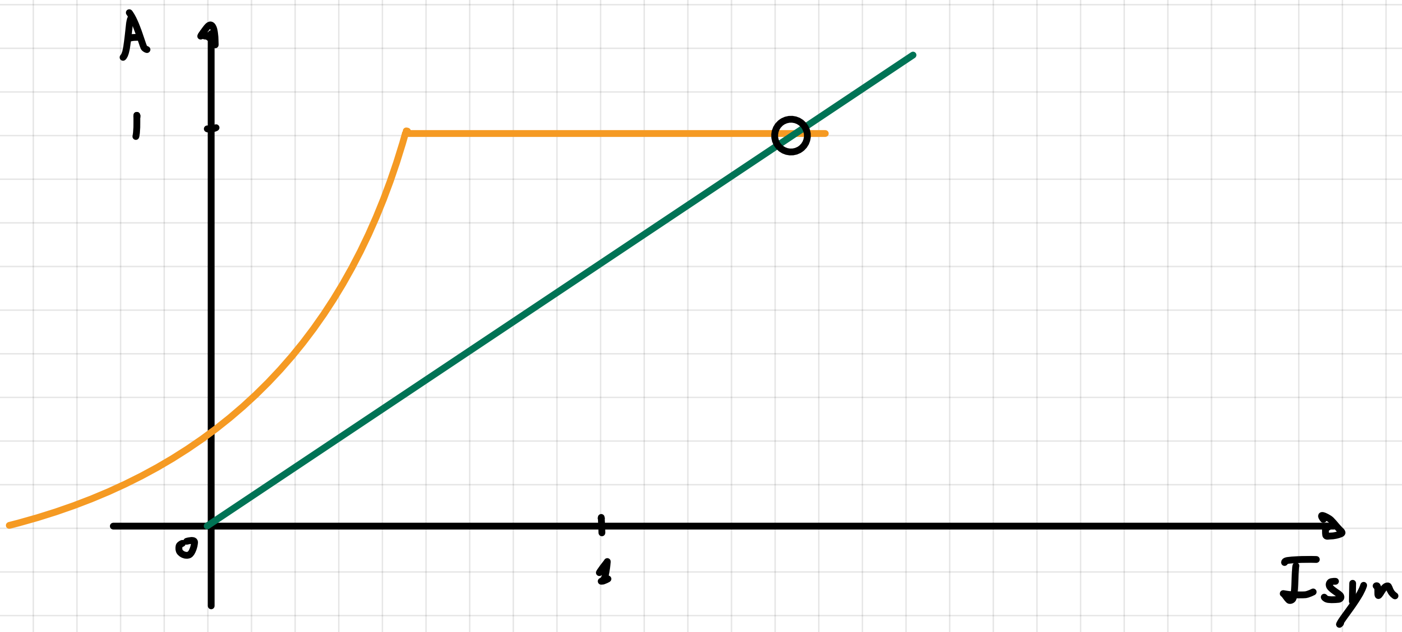
$$A = g(I)$$

Here is the plot!



c) Three fixed points (Two stable, one unstable).

Adding external current shifts the orange curve left:



Only one stable point remains!
the stable point persists if Iext is even higher.

d) → the green line becomes less sloped.
the high-valued fixed point shifts right.

→ the green line becomes so sloped
only one fixed point remains in 0

Exercise 3

a) Each opening deposits

$$q = \int_0^\infty (V_0 - E)\, g(t)\, dt =$$

$$= (V_0 - E)\, g_0\, t_d$$

Hence

νt_d is also the number of channels open at any given time

$$\langle I \rangle = \nu (V_0 - E)\, g_0\, t_d$$

b) At any time, the number of open channels is distributed as

$$O \sim \text{Pois}(\nu t_d)$$

whose variance is νt_d. Hence

$$\text{Var}(I) = \text{Var}\left((V_0 - E)\, g_0\, O\right) =$$

$$= (V_0 - E)^2 g_0^2 \, \text{Var}(O) =$$

$$= (V_0 - E)^2 g_0^2 \, \nu t_d$$

c) The number of openings / closings that happens in an interval of time τ are distributed as

No !! Closings depend on $I(t)$!

$$S^+, S^- \sim \text{Pois}(\nu\tau)$$

Identically and independently distributed!

As a result we have

$$I(t+\tau) = I(t) + \underbrace{S^+ - S^-}_{\sim 0}$$

Hence

$\langle S^+ - S^- \rangle = 0$

$$\langle I(t) I(t+\tau) \rangle =$$

$$= \langle I(t) \left(I(t) + S^+ - S^- \right) \rangle =$$

$$= \langle I(t)^2 + I(t) \left(S^+ - S^- \right) \rangle =$$

$$\langle I(t)^2 \rangle + \langle I(t) \left(S^+ - S^- \right) \rangle$$

NOPE! Revisit this later!

Exercise 4

a)

$$\nu(t) = \nu_0(E) + \left(\nu_0(U) - \nu_0(E)\right)\left(1 - e^{-\frac{(t-t_0)}{\tau}}\right)$$

b) Because $T \gg \tau$, $\nu(t)$ converges to $\nu_0(U)$ very early in the observed window.

→ Time between closing and next opening is thus distributed exponentially in $\nu_0(U)$:

$$f(t) = \nu_0(U)\, e^{-\nu_0(U)\, t}$$

→ Time between consecutive openings requires a shift by t_d:

$$f(t) = \begin{cases} \nu_0(U)\, e^{-\nu_0(U)(t - t_d)} & \text{if } t \geq t_d \\ 0 & \text{otherwise} \end{cases}$$

c) Between two consecutive openings the channel spends:

- τ_d time open

- An amount of time closed that is exponentially distributed in $\frac{1}{\tau_d}$.
  Expected value: τ_d.

As such, the channel will be open (approximately) <u>HALF</u> of the time.